# Design and Optimization of Airline Network

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**Abstract:** The optimized design of an airline network means arranging the specific transportation paths for all O-D flows in the network composed of the connected lines between cities involved in air transportation with minimizing the total transportation cost as the purpose. Different O-D flow transportation methods will form different airline network structures. No matter what network structure the airline chooses as its own airline network, when the airline service scope becomes very large, manual calculation will not help determine the optimal airline network structure and mathematical models and computer solvers will be required to assist the decision-making. The design and optimization of an airline network is very essential to any high-revenue, high-cost and low-profit airline.

## 1. Introduction

On the design of airline networks, a lot of researches have been done both at home and abroad. Researches started earlier abroad and have also achieved a lot of results. O'Kelly (1987) constructed the integer programming model for hub-and-spoke airline networks for the first time and designed two heuristic algorithms. Klincewicz (1991) proposed using the exchange-based heuristic method to solve the p-hub median problem, and using the nearest point assignment method to get the initial solution. Klincewicz (1992) used the tabu search method and greedy random search procedure to solve the p-hub address problem and achieved a good solution. Based on the research result obtained in 1987, O'Kelly (1992) took into account the setup cost of hub and re-established the hub address model, and solved the upper and lower limits of the problem through an algorithm. Skorin-Kapov and Skorin-Kapov (1994) proposed a heuristic algorithm based on tabu search also to address the problem raised by O'Kelly (1987), avoiding unnecessary loop computations and optimizing the solution. The Campbell (1994) proposed a mixed integer programming model for the hub median problem for the first time, which reduced the difficulty of calculation. Aykin (1994) used the Lagrangian relaxation method to study the optimization of hub networks with capacity constraints and achieved satisfactory results by carrying out calculation with samples from 40 cities. Aykin (1995) classified the hub-and-spoke airline network design into two categories: non-strict hub policy and strict hub policy, and gave specific algorithms for the two models, and compared the two models with the samples from air transport in the U.S. Campbell (1996) established a path decision variable model for the p-hub median problem, and gave two high-quality heuristic algorithms MAXFLO and ALLFLO. Ernst and Krishnamoorthy (1996) used the simulated annealing algorithm to study the median problem of uncapacitated single p-hub.

## 2. Problems to Be Solved In the Airline Network Design

Airline network design is planning based on airline network analysis. It is mainly to solve the problems regarding three routes. The first is about the OD flow route or the passenger/cargo transport route; the second is about the airplane route; and the third is about the aircrew route. Only when the airline completes the design of the three routes can it execute specific transportation tasks and finally solve air transportation problems.



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In general, when there is a transport route between two nodes, the same airplane and crew do not necessarily always correspond to the passengers. These three routes are often interactive but do not overlap in space, as shown in Figure 1. A1, A2 and A3 are airplanes and C1 and C2 for crews. If A1 is delayed, and the crew executing the route B→E is C1, then A3 has to wait for crew C1 at node B, making it impossible to complete the transport task on B→E as scheduled.

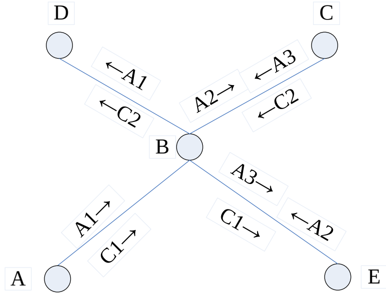


Figure 1 Relations Between Three Routes in Airline Network Design

The three routes are intertwined and interactive with each other and do not overlap, making air transport extremely complicated. In reality, an airline needs to take these three routes into overall account in the airline network analysis, but airline network design only solves the OD flow problem. If it does not solve the airplane and crew routes, the airline cannot determine the airplanes and crews for specific air routes, which will further affect the completion of the whole air transport tasks. To address this problem, an airline often designs its airline network based on the following two scenarios:

①Airline network design with a specified structural form. For example, an airline has decided to use a point-to-point route network structure or hub-and-spoke airline network structure, so it determines the specific OD flow route between any two nodes through multiple qualitative and quantitative researches.

②Airline network design without a specified structural form. By this time, the airline has not determined the airline network structure, and whether it will execute a point-to-point route network structure or hub-and-spoke airline network structure or a mixed airline network involving both point-to-point and hub transfer will totally be subject to the simulation and optimization result. The airline will execute whatever optimized routes and network it is given.

Airline network design should be performed based on airline network analysis and feasibility analysis. This includes the OD pair market and traffic size the network can serve, unit flow cost and capacity constraint as well as the investment to improve or reconstruct the network. Whether with or without a specified structural form, airline network design should always solve the following four problems first. As shown in Table 1.

Table 1 Problems to be solved first in airline network design.

| Problems to be solved first in airline network design | Content |
|---|---|
| Service market and demands | Which OD-pair markets to serve and what traffic size |
| Transportation cost | Unit flow cost, i.e. the cost of unit flow |
| Airline and airport capacity | With or without any capacity constraint. No capacity constraint is deemed as infinite capacity. |
| Input | Investment used to improve or reconstruct the airline network |

In general, the service markets and demands can be addressed through market research and demand forecast; in other words, these are already completed in the marketing plan.

Transportation cost can be determined through historical data analysis. If there is no historical data, it is often obtained by the analogy method. For example, there are a variety of airplane types to choose from, and different types correspond to different unit flow costs. If no specific airplane type is arranged in the airline network design phase, such data cannot be directly obtained. Instead, an expected value should be forecasted by the averaging method and then analysis should be carried out based on it.

The capacity of a route or an airport is forecasted based on the published air transport data or the time when it may be available. The capacity here is not the total capacity of the airport but the capacity that can be allocated, because an airline only designs its own airline network. For example, if the capacity of Pudong Airport is 900 flights/day and an airline is assigned with 100 flights/day, so the airline should set the capacity at 100 instead of 900 in the airline network design. If the airline has not been assigned with any capacity, then it has to make a forecast of how many flights per day in the airline network design.

Input refers to the investment that an airline makes in order to improve or rebuild its airline network. In order to enhance its competitiveness and expand its air transport market share, in addition to fare discount and other preferential policies to attract passengers, what an airline needs most is to improve its own quality, such as making investments to optimize its network structure and improving its flight schedule and capacity allocation.

## 3. Airline Network Optimization Model

### 3.1 Optimization model for general airline networks

The optimized design of the airline network with no specified structural form can be represented by the Magnanti universal model, as shown in the type of (1) to (5).

$$\min Z(X,Y)=\sum_{(i,\ j)\in A}\sum_{\{k,\ m\}\in A} W_{ij}\left(C_{km}x_{km}^{(i,\ j)}+C_{mk}x_{mk}^{(i,\ j)}\right)+\sum_{\{k,\ m\}\in A} F_{km}y_{km} \tag{1}$$

$$\text{s.t.} \sum_{k\in N} x_{km}^{(i,\ j)}-\sum_{t\in N} x_{mt}^{(i,\ j)}=\begin{cases}-1,\ m=i;\\ 1,\ m=j;\\ 0,\ others。\end{cases} \quad \forall(i,j)\in A, k\in N \tag{2}$$

$$\sum_{(i,j)\in A} W_{i,j}x_{km}^{(i,j)} \le D_{km}y_{km}, \quad \forall(k,m)\in A \tag{3}$$

$$\sum_{\{k,m\}\in A} F_{km}y_{km} \le B \tag{4}$$

$$x_{km}^{(i,\ j)}, x_{mk}^{(i,\ j)} \ge 0, \ y_{km}\in\{0,1\}, (i,\ j)\in A, \{k,\ m\}\in A \tag{5}$$

In order to understand this model, we should first understand the meaning of each parameter in the model. The subscript (i, j) denotes the OD pair, where the parentheses indicate the direction, i.e. i→j; $W_{ij}$ represents the traffic between node i and node j, i.e. the OD flow demand; k, m, t are general nodes; the set A represents the set of sides (air routes), which also includes the OD pair; N represents the set of nodes; {k, m} represents the side km, where the brackets indicate no direction; the parameter $C_{km}$ represents the cost per unit flow passing along the side km, that is, the cost per unit flow on (k, m); $F_{km}$ is the construction cost of (k, m); $D_{km}$ is the capacity of (k, m), which, in this model, only involves the capacity of sides and does not include the capacity constraints of nodes; B is the total investment; the variable $x_{km}^{(i,j)}$ represents the component of flow, which is a variable of flow, expressed in a proportion, a percentage or a decimal; $W_{ij}x_{km}^{(i,j)}$ stands for the flow of the OD pair (i, j) on the side (k, m); $y_{km}$=0,1, which is a 0-1 variable, indicating whether the side (k, m) is under construction, in service or selected. If this route is about to be built, this variable will be 1; otherwise it will be 0.

The optimization model for general airline networks was proposed by Magnanti in 1987. In the objective function, $W_{ij}x_{km}^{(i,j)}$ is multiplied by unit flow cost $C_{km}$ to represent the total cost corresponding to the flow of the OD pair (i, j) on the side $(k,m)$; $W_{ij}x_{km}^{(i,j)}$ multiplied by $C_{mk}$ is the total cost of the flow in the reverse direction.

$\sum_{(i,j)\in A}\sum_{(k,m)\in A} W_{ij}(C_{km}x_{km}^{(i,j)} + C_{mk}x_{mk}^{(i,j)})$ in the objective function is the total flow cost; in the objective function, $F_{km}$ is the construction cost, $y_{km}$ stands for whether construction is required, so the sum of their products stands for the total network construction costs. The minimization of the sum of the flow cost in the network and the total network construction cost is just the objective function; in other words, the objective function is the minimum sum of the network flow cost and network construction cost.

The first constraint in the model is the flow balance constraint, which is the restriction on network flow balancing. For the intermediate nodes, the inflow is equal to the outflow; for the initial node, there is only outflow, and for the terminal node, there is only inflow. Here m is the free subscript and (i, j) is an OD pair. Under this constraint, i, j and m are composite numbers, and when they take different values, there will be different constraints. Assuming that the free subscript m is i, it will be the initial node of the OD pair; if m is j, then it will be the terminal node of the OD pair. From i to j, the traffic flows in and out, and their values are -1 and 1, respectively; $x_{km}^{(i,j)}$ is a component of flow, which is a proportion. If the sum of such components is 1, it means all the volume $W_{ij}$ needs to be transported out; otherwise, if it is less than 1, then part of the volume is not transported out and not all the transport demands are met.

The second constraint in the model is the capacity constraint. Here the capacity is not limited by the number of flights, but the size of traffic. For passengers, the capacity is the number of passengers to transport; for freight, the capacity refers to the quantity of goods (tons). $\sum_{(i,j)\in A} x_{km}^{(i,j)}$ represents the sum of the flows of all OD pairs on the side km. When the side km is constructed, it cannot exceed $D_{km}$.

The third constraint in the model is the investment constraint. $\sum_{(k,m)\in A} F_{km}y_{km}$ is the total investment by the airline, which cannot exceed B.

The order and the number of variables in the airline network optimization model have something to do with the sparseness of the network. In the fully connected network configuration, in the above model, there are $n^3$ Category 1 constraints, because there are $n^2$ OD pairs and n nodes; there are $n^2$ Category 2 constraints; and there is one Category 3 constraint. Therefore, the total order of magnitude of the model is $n^2$; in the most severe case, there are $n^4$, equivalent to the number of OD pairs ($n^2$) multiplied by the number of sides ($n^2$).

### 3.2 Optimization model for hub-and-spoke airline networks

The modeling of hub-and-spoke airline network should first consider cost. Transportation cost and airplane type are related, but at this time it is not known how a type of airplanes are distributed in the network. This will be determined only during flight scheduling.

The second consideration is the confluence of hub operations. Through hub transit, the traffic from a number of branches can be gathered for centralized operations so as to achieve scale effect, as shown in Figure 2.

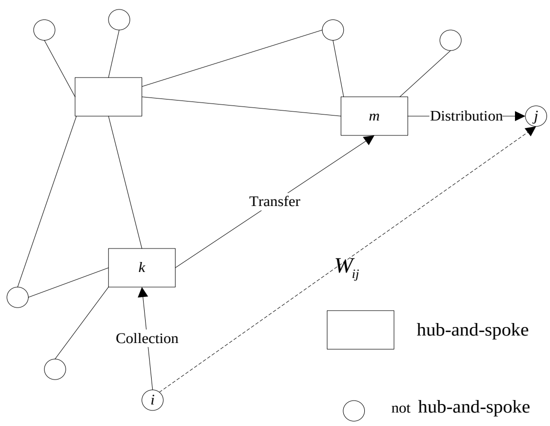


Figure 2 Diagram of transit transportation

There are 3 hubs. The traffic from i through branch ik may head towards other nodes, so node k has the confluence effect; in other words, the traffic is transited through node k to other nodes. The traffic volume at node k is not just the flow from i to k, but also the confluence result of multiple side flows, constituting large-scale transportation, so it has the economies of scale, especially from hub node to hub node. For example, from k to m, the economies of scale will be more significant, and from m to j, there will also be some economies of scale. In Figure 2, the section from i to k is called confluence, that from k to m is transit, and that from m to j is shunting. So $W_{ij}$ from i to j can get to the destination through one confluence, one transit and one sub-carriage.

If the scale economy is not taken into account, the cost $C_{ij}$ from i to j will be equal to the sum of $C_{ik}$ from i to k, $C_{km}$ from k to m and $C_{mj}$ from m to j, i.e. $C_{ij}=C_{ik}+C_{km}+C_{mj}$. If the scale economy is considered, each cost item is weighted first and then added up. For example, each cost item is multiplied by a factor of less than 1. This approach is very subjective, but there is no other better way to simplify the operation. In general, the scale economy between hub nodes should be the largest, denoted as α, that on the confluence side is denoted as γ, and that on the shunting side is denoted as β, and α, β and γ have the following relationships: $0<\alpha\leq\gamma\leq\beta\leq1$. At this time, the cost $C_{ijkm}$ from i to j can be expressed as: $C_{ijkm}=\gamma C_{ik}+\alpha C_{km}+\beta C_{mj}$, which represents the cost of unit flow from i to j through k and m.

As the specified network is hub-and-spoke airline network, $x_{ijkm}$ represents the component transported from i to j through hub nodes k and m; $W_{ij}x_{ijkm}$ means the part of the traffic between i and j transited through k and m; $W_{ij}x_{ijkm}C_{ijkm}$ represents the cost of the traffic volume.

According to the above rules, if any OD pair transits through at least 2 hubs, there is no capacity constraint on nodes and sides, and the number of hub airports is specified as p, the corresponding mathematical model for the minimized total transportation cost of hub-and-spoke airline network is denoted as UMpHMP (Uncapacitated Multiple Hub Middle Problem) optimization model, as shown in the type of (6) to (11):

$$Z^{*}=\min Z(y,X,s)=\min\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{m=1}^{n}W_{ij}C_{ijkm}x_{ijkm} \tag{6}$$

$$s.t. \qquad \sum_{k=1}^{n}y_k=p \tag{7}$$

$$\sum_{k=1}^{n}\sum_{m=1}^{n}x_{ijkm}=1, i, j=1,2,\dots n \tag{8}$$

$$\sum_{m=1}^{n}x_{ijkm}\leq y_k, i, j, k=1,2,\dots n \tag{9}$$

$$\sum_{k=1}^{n}x_{ijkm}\leq y_m, i, j, m=1,2,\dots n \tag{10}$$

$$y_k\in\{0,1\}, k=1\dots n; x_{ijkm}\geq 0, i, j, k, m=1,2,\dots n \tag{11}$$

This kind of design problem involves two kinds of variables - one is the flow variable and the other is the design variable. The design variable needs to choose hubs rather than sides. As long as the hub nodes and $x_{ij}$ are fixed, the specific connections between OD pairs will be obtained and the sides and the whole structure of the network can be determined. At this time the design variable is not $y_{km}$ but $y_k$, which is a 0-1 variable, indicating whether node k is selected as the hub airport. Suppose node B, C and D are hub airports, node A and E are any two non-hub node airports. By solving the above model, we obtain the value of $x_{ijkm}$ and then determine the structure of the airline network. For example, $x_{AEDC}=1$, meaning that the route between non-hub node A and the non-hub node E is A→D→C→E; and if $x_{AEDC}=0.5$ and $x_{AEB}=0.5$, then there are two transport routes between node A and node E - A→D→C→E and A→B→E, as shown in Figure 3.

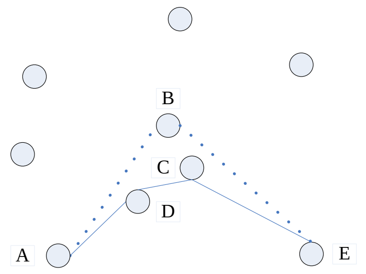


Figure 3 Transport routes between nodes

Assuming that node B, C, D and F are hub airports, that node A and E are any two non-hub node airports and that the routes between (A, E) are A→D→C→E, A→B→C→E and A→B→F→E, then $x_{AEDC}+x_{AEDC}+x_{AEDC}=1$, as shown in Figure 4.

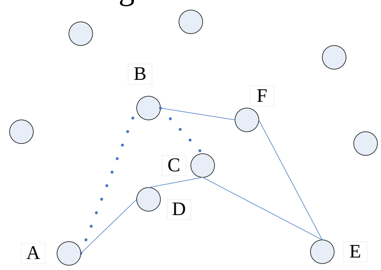


Figure 4 Transport routes for OD pairs and flow distribution

Constraints (1-9) and (1-10) indicate that only when nodes k and m are hub airports, the traffic at i and j can be transported through them. Their purpose is to force transports between all OD pairs to transit through hubs and prohibit direct transport between the feeder airports. The strict hub constraint is implied here.

For hub-and-spoke airline network optimization models, in addition to the UMpHMP model, there are also tge strict CMAHLP model with capacity constraint, no specified number of hubs, multiple allocations (Capacitated Multiple Allocation Hub Location Problem), the strict USpHLP model with no capacity constraint, specified number of hubs and single allocation (Uncapacitated Single Hub Location Problem), the strict CSpHLP model with capacity constraint, specified number of hubs and single allocation (Capacitated Single Hub Location Problem) and other capacitated models.

## 4. Conclusion

This paper first compares and analyzes the performance of the point-to-point airline network and the hub-and-spoke airline network and reveals the features of these two different network structures.

Second, based on the airline network analysis, this paper analyzes the airline network design method when the structural form is specified or not specified and points out the problems to be solved first in airline network design.

Then, by using the optimization model for general airline networks, this paper focuses on discussing the optimized design of airline networks with specified structures, analyzes the mathematical models for the optimized design of hub-and-spoke airline networks that are with or without restriction on the number of hubs, capacity or incapacitated, single-allocation or multiple allocation and strict or non-strict, laying a solid foundation for subsequent research.

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